The Role of Social Networks on Regulation in the Telecommunication Industry

by

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Abstract

In this paper we provide a model to study the equilibrium behavior in a telecommunication market under a Calling Party Pays regime (CPP), where two interconnected firms compete in the presence of social networks among customers. In the analysis we consider that each customer makes optimal market participation and affiliation decisions, leading to two different forms of network externalities. On the supply side, we consider price competition under both, linear and non-linear pricing schemes. In particular, we consider different meaningful forms of price discrimination as on-off net price discrimination or two part tariffs. A first result is that social structure matters because equilibrium prices and welfare critically depend on social network structures. Then we use our model to study the impact of alternative regulatory interventions under different pricing schemes, and we show that policies oriented to reduce transportation costs are more effective than those oriented to bound interconection charges.

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1 Introduction

In the last decades a growing literature have been focused on social interactions modeled through the use of network structures or graphs, where agents are represented by nodes and their relationships are represented by arcs between those nodes. These network structures play an important role in many economic situations and have been studied widely. Armstrong (2002), Jackson (2010) provides an outstanding summary of theory and applications. However, as far as we know, no other authors have studied the interconnection problem between telecommunication operators considering that users interact through a social network.

The main characteristic related to the social network environment is that the number of calls (or direct conections) between any pair of consumers depends not only on prices and the level of differentiation in services, but also on how socially close they are in the network. This context would be relevant in order to determine a more realistic market performance and also how regulation should be accomplished. Over the last years several articles have been focused on the study of the equilibrium interconnection strategies in telecommunication markets, in a framework where heterogeneity of consumers is recognized (see for example Dessein, 2004 and Hahn, 2004, among others). These approaches have represented a significant improvement in the effort to obtain models more closely related to reality. However, the social network structure among consumers has been mostly ignored, and heterogeneity has been usually motivated on the grounds of different propensities to make calls.

Several papers are closely related to this article. The seminal ones are Laffont *et al.* (1998a,b) and Armstrong (1998). The equilibrium behaviour of interconnected firms in the presence of heterogeneous consumers has been analyzed by Dessein (2004), Hahn (2004), Hurkens and Joen (2009), Hoernig *et al.* (2011), Jullien *et al.* (2013), among others. For example, Cambini and Valletti (2008) proposed a model of information exchange where the closer the calling parties are in social terms, the higher the intensity of information exchange. However, the use of social networks to model the connections among consumers has been introduced by Harrison *et al.* (2006) in a context of linear pricing schemes with a focus on a comparison between a regulated and an unregulated environment.

In the current article, however, our main goal is to develop a model that allow us to study the economic impact of more general and meaninful pricing schemes in the presence of social networks. For example, two very important nonlinear pricing schemes that are usual in telecommunication markets are: First, when firms can price discriminate depending on the destiny of a call (on-off net price discrimination), and second, when two part tariffs are feasible (for example when equipment is subsidiazed). In the modelling strategy we borrow some tools from Operation Research, in order to get a model flexible enough to study a broad variety of pricing schemes and regulatory environments, as the ones mentioned above, all of this under the constraint that the agents involved, firms and consumers, make optimal decisions in equilibrium.

Finally, we apply the model to study the efectiveness of two alternative regulatory interventions under different pricing schemes. First, a regulatory intervention based on setting access charges below marginal costs would produce a positive impact on competition, reducing equilibrium prices to consumers. Second, alternative policy interventions, represented by reductions in transportation costs, also intensify competition when services are differentiated. Interestingly, our results show that a regulatory intervention focused on reducing access charges below marginal costs enhances welfare.[1] However, an alternative policy intervention equivalent to reducing transportation costs is much more effective in getting closer to efficient outcomes. Welfare also increases when the social network is more dense, but this characteristic of the network can not be subject to policy implications.

The rest of the paper is organized as follows: In section 2 we develop the basic economic model, including the agent's demand, the firms' problem and the general game played by the two firms. In section 3 we specialize the model to study the equilibrium effects under linear non-discriminatory pricing and under two forms of nonlinear pricing. In section 4 we provide and example of application in regulation illustrating the main results under the different pricing schemes. Finally, the conclusions are stated in section 5.

2 The Economic Model

In the model we assume the existence of a social network, represented by a graph g. Nodes in the graph represent agents (indexed by $i \in I$) and a link between a pair of agents represent a social connection between them. The graph g is generated using random regular graphs (see Bollobas, 2001), where the *connectivity degree* d of graph g represents the average number of social connections accross agents.

There are two firms, A and B, offering horizontally differentiated communication services (for example two wireless companies) and consumers have to decide whether they participate at all in the market and, in such a case, which firm to subscribe to. In order to make the affiliation decision, agents take into account the pricing schemes offered by each firm and her own preferences for the services provided. On the other hand, the preferences are modeled in a similar way to a standard Hotelling horizontally differentiated model: each agent i in the social network (i.e. each node in g) is endowed with a realization of a taste variable x_i, randomly assigned from a uniform grid with support in $[0, 1]$. In what follows we assume that firm A is "located" in 0 and firm B in 1. None of them provide the "ideal service" to agent i, positioned in x_i, unless x_i itself be zero or one.

[1] The welfare increase comes from a reduction in oligopoly rents rather than calling externalities, which are not modeled here.

2.1 The Agent Demand

Consider the affiliation decision problem of agent i. If agent i decides to subscribe network $l = A, B$ then we will say that she belongs to the set $I_l \subseteq I$ of subscribers to l. Agent i's demand for calls is represented by the vector $q_i = (q_{ij})_{j \in I, j \neq i}$, where the generic element q_{ij} is the number of calls that agent i makes to agent j. Then the gross utility of agent i can be described as follows:[2]

$$U_i(q_i) = \sum_{j \in I, j \neq i} \delta^{t_{ij}} u(q_{ij}) \qquad \text{with} \qquad u(q_{ij}) = \frac{q_{ij}^{1-1/\eta}}{1 - 1/\eta} \tag{1}$$

where:
$\delta :$ represents a discount in utility when agent i calls other agents located one step farther in the network g. Accordingly, it satisfies $0 < \delta < 1$.
t_{ij} : it is the shortest distance (in terms of links) connecting agents i and j. We consider $t_{ij} = 0, 1, 2, ...$ so that if the agents are direct neighbors, the discount factor is $\delta^0 = 1$. On the other hand if the agents i and j are not connected then $t_{ij} = \infty$.
$\eta :$ is a constant parameter representing the elasticity of demand, which is assumed to be constant, greater than 1 and independent of j. [3]

A typical and general pricing scheme applied for firm A (analogous for B) is given by $T(q_A, \widehat{q}_A) = F_A + p_A q_A + \widehat{p}_A \widehat{q}_A$ where F_A is a fixed charge and p_A is the price per call for a subscriber in network A when she is calling another subscriber in network A (on net call), while $\widehat{p}_A$ is the price per call for a subscriber in network A when she is calling a subscriber of B (off net call). The notation q_A and $\widehat{q}_A$ refers to the corresponding levels of on and off net calls, respectively.

The model is flexible enough to consider such a general pricing scheme, but in this article we focus on the impact of some particular but empirically relevant schemes. First, we study a simple linear non-discriminatory pricing scheme (case 1) where $F_A = 0$ and $p_A = \widehat{p}_A$ (analogous for B). Then we consider two types of nonlinear pricing schemes (cases 2.a and 2.b). In case 2.a, we study the role of price discrimination depending on the destiny of a call (on-off discrimination), but we set F_A and F_B as equals to zero and

[2] Note that we are assuming in this formulation that all the individual in the network can receive calls even if she is not affiliated to A or B. This assumption is made for tractability but it is not so demanding if we consider that a prepaid phone can always receive calls in a CPP regime. In what follows, however, we will discuss what happens if we depart from this assumption.

On the other hand, we are not considering *calling externalities*, i.e. customers do not get utility from incoming calls. The introduction of these kind of externalities is direct, however, through the addition of a term $v(q_{ji})$.

[3] Note that this assumption is consistent with the empirical literature. See for example Hazlett and Muñoz (2009).

finally, in case 2.b, we study the role of two part tariffs, but we impose $p_A = \widehat{p}_A$ and $p_B = \widehat{p}_B$.

For practical reasons, we are assuming that a disconnected individual can receive calls (for example in the fixed network), and in such a case, the call is considered on net.

Suppose that after observing the price schemes offered by the firms, agent i has to decide which firm to affiliate or to remain out of the market. In order to make that decision, she needs to figure out her net consumer surplus in the case that affiliate each firm. If she decides to affiliate firm A, the vector of calls $q_i = (q_{ij})_{j \in I, j \neq i}$ to all her contacts in the network g is defined by:

$$W_i(p_A, \widehat{p}_A) = \max_{q_i} \left\{ U_i(q_i) - p_A \sum_{\substack{j \neq i \\ j \in I \setminus I_B}} q_{ij} - \widehat{p}_A \sum_{j \in I_B} q_{ij} \right\} \tag{2}$$

Solving this maximization problem, we obtain his/her demand's components:

$$q_{ij}(p) = \left(\frac{p}{\delta^{t_{ij}}} \right)^{-\eta} \quad \text{with} \quad p = p_A, \widehat{p}_A \tag{3}$$

Intuitively, for the same price p, agent i makes more calls to contacts located closer in the social network g than to those farther in it. Moreover, the possibility to discriminate depending on the destiny of a call makes the number of calls depending also on where agent j is affiliated. Therefore, plugging into equation 2 we get the indirect utility function:

$$W_i(p_A, \widehat{p}_A) = \sum_{\substack{j \neq i \\ j \in I \setminus I_B}} \delta^{\eta t_{ij}} \frac{p_A^{1-\eta}}{\eta - 1} + \sum_{j \in I_B} \delta^{\eta t_{ij}} \frac{\widehat{p}_A^{1-\eta}}{\eta - 1} \tag{4}$$

and an analogous result arises for firm B.

Consider the parameter τ representing the unit cost associated to the fact that agent i, located in x_i, has to subscribe to network A located in 0 or network B located in 1. None of them provide the "ideal service" (this would be the case if some network were located precisely in x_i) so the cost of selecting a service different from i's preferred one is assumed to be $x_i \tau \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$ if agent i selects network A or $(1 - x_i)\tau \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$ if network B is preferred. It is important to note that in this model we assume that agent i incurs in a discounted disutility for calls due to the imperfect matching between her preferences and the service provided, where the discount appears because the imperfection is more annoying the closer is agent j to i in the social network. The total cost of imperfect matching is the sum of all the pairwise discounted costs. In addition, note that the cost to agent i of an imperfect service to call agent j is assumed independent of the number of calls.[4]

Let us define the net surplus for consumer i when affiliates to firm l (A or B) as:

$$w_i(p_l, \widehat{p}_l, F_l, d_i^l) \equiv W_i(p_l, \widehat{p}_l) - F_l - \tau d_i^l \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$$

where d_i^l is the distance between consumer i and firm l. The preference for A or B depends on whether x_i is to the right or to the left of a critical value x_i^* given by:

$$w_i(p_A, \widehat{p}_A, F_A, x_i^*) = w_i(p_B, \widehat{p}_B, F_B, 1 - x_i^*)$$

If $x_i < x_i^*$, (resp. $x_i > x_i^*$) agent i prefers network A (resp. B) even considering that network A does not provide her the ideal service (and has to pay $\tau x_i \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$ by the imperfect matching). Solving for x_i^*, we got:

$$x_i^* = \frac{1}{2} + \sigma_i \left[W_i(p_A, \widehat{p}_A) - F_A - (W_i(p_B, \widehat{p}_B) - F_B) \right] \qquad \left(\text{with} \quad \sigma_i = \frac{1}{2\tau \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}} \right)$$

Let us define $\alpha_i = 0$ if agent i prefers network A and $\alpha_i = 1$ if agent i prefers network B.

Accordingly, the incentive compatibility constraint can be written as:

$$\alpha_i = \begin{cases} 0 & \text{if} \quad x_i < x_i^* \\ 0 \text{ or } 1 & \text{if} \quad x_i = x_i^* \\ 1 & \text{if} \quad x_i > x_i^* \end{cases} \tag{5}$$

However, we also have to consider the option to remain disconnected. Agent i affiliates to telecommunication services (one of the two firms) if and only if

[4] Alternative approaches would be to make the transportation cost dependent on the utility obtained from the calls or dependent on the number of calls. Our selection is consistent with Laffont *et al.* (1998a).

$$Max\left\{w_i(p_A,\widehat{p}_A,F_A,x_i),w_i(p_B,\widehat{p}_B,F_B,1-x_i)\right\}\geq 0$$

equivalently, we can define:

$$\Omega_i(p_A,\widehat{p}_A,p_B,\widehat{p}_B,F_A,F_B,x_i,\alpha_i)=(1-\alpha_i)w_i(p_A,\widehat{p}_A,F_A,x_i)+\alpha_i w_i(p_B,\widehat{p}_B,F_B,1-x_i)$$

and then, the individual rationality constraint for agent i is modeled by β_i such that:

$$\beta_i=\begin{cases}0 & \text{if} \quad \Omega_i<0\\ 1 & \text{if} \quad \Omega_i\geq 0\end{cases} \tag{6}$$

Accordingly, for example, in order that agent i affiliates firm A, it is necessary that she prefers A to B ($\alpha_i=0$) and that the net surplus from the affiliation to A be no negative ($\beta_i=1$).

2.2 The Firm's Problem

Assume that each firm pursues maximization of profits and that Calling Party Pays (CPP) is the interconection regime. When access charges are given by a_A and a_B, firm A (resp. B) will select its prices $p_A,\widehat{p}_A,F_A$ (resp. $p_B,\widehat{p}_B,F_B$) such that:[5]

$$\begin{gathered}\max_{p_A,\widehat{p}_A,F_A\geq 0}\ \pi_A(p_A,\widehat{p}_A,p_B,\widehat{p}_B,F_A,F_B,a_A,a_B)=\\ \sum_{i\in I_A}\left\{\sum_{\substack{j\in I\setminus I_B\\ j\neq i}}q_{ij}(p_A)(p_A-c_A^o-c_A^f)+\sum_{j\in I_B}q_{ij}(\widehat{p}_A)(\widehat{p}_A-c_A^o-a_B)+F_A-f\right\}+\\ \sum_{i\in I_B}\sum_{j\in I_A}q_{ij}(\widehat{p}_B)(a_A-c_A^f)\end{gathered} \tag{7}$$

where:[6]
f : is the fixed cost incurred by a firm when it affiliates a new subscriber.
c_A^o : is the cost of originating a call for firm A (c_B^o is defined analogously).
c_A^f : is the cost of terminating or finishing a call for firm A (c_B^f is defined analogously).

[5] Note that we have assumed that individuals not affiliated to firms can be called incurring in marginal termination costs.

[6] In what follows, when we solve an optimization problem, we always assume that g, f, c_A^o, c_B^o, c_A^f, c_B^f, $\{x_i\}_{i=1}^I$ and τ are all given exogenously.

a_A : is the access charge that firm A applies to firm B in order to terminate a call from a subscriber of B to a subscriber of A (a_B is defined analogously).

The structure given in problem (7) is not convenient, because the individual rationality and incentive compatibility constraints for customers are embeded in the sets where the sums are calculated. In what follows we want to make explicit those constraints to facilitate the algorithm to find the Nash equilibrium prices in the competition between firms. In order to do so, our goal will be to express both constraints in linear form, so as to write firm A's problem as:

$$\underset{p_A,\widehat{p}_A,F_A\geq 0}{Max}\ \pi_A(p_A,\widehat{p}_A,p_B,\widehat{p}_B,F_A,F_B,a_A,a_B;\alpha,\beta) \tag{8}$$

$$\begin{aligned} s.t. \quad & H_1\alpha \leq z_1, \quad \alpha \in \{0,1\}^I \qquad (IC \text{ constraints}) \\ & H_2\beta \leq z_2, \quad \beta \in \{0,1\}^I \qquad (IR \text{ constraints}) \end{aligned}$$

With this goal in mind, we separate the problem in two parts. First, we write participation and affiliation decisions as a system of inequality constraints and then, we write the objective function as in (8). Even so, the dimension of the problem makes it unsolvable in the general case, so we focus on three particular schemes: a simple linear non discriminatory pricing scheme (case 1) and then two types of nonlinear pricing schemes (cases 2.a and 2.b).

3 The Model under Relevant Pricing Schemes

3.1 Case 1: Linear Non Discriminatory Prices

In order to simplify this case we can assume that for all the consumers the individual rationality constraint is not binding, i.e., $\beta_i = 1\ \forall i \in I$. Otherwise, adding a constant to the utility function is enough to satisfy this condition. Then firm A's problem can be written as:

$$\underset{p_A\geq 0}{Max}\ \pi_A(p_A,p_B,a_A,a_B;\alpha) \tag{9}$$

$$s.t. \quad H_1\alpha \leq z_1, \quad \alpha \in \{0,1\}^I \qquad (IC \text{ constraints})$$

Note that the previous structure is not warranted in general, because we are requiring that affiliation decisions be represented by a linear constraint. The gains from obtaining such a neat representation of the problem are very important. First, despite of the introduction of social networks, and the requirement that everybody makes optimal decisions, the problem is kept simple; second, we will be able to expand the set of situations where the model applies without changing the structure, and third, it helps us to find an algorithm to solve it. With this goal in mind, we separate the problem in two parts. First, we need to write the vector of optimal affiliation decisions as the solution of a linear inequality constraint ($H_1\alpha \leq z_1$, $\alpha \in \{0,1\}^I$) and then, we have to write the objective function as in (9), so that we make explicit the dependance of the objective function on the vector of affiliation decisions α. The following sections are devoted to these tasks.

3.1.1 The Constraint

Using the definition of α_i the optimal affiliation decision can be written as:

$$\alpha_i = \begin{cases} 0 \text{ if } & x_i < x_i^* \\ 0 \text{ or } 1 \text{ if } & x_i = x_i^* \\ 1 \text{ if } & x_i > x_i^* \end{cases}$$

where

$$x_i^* = \frac{1}{2} + \sigma_i \frac{(p_A^{1-\eta} - p_B^{1-\eta})}{\eta - 1} \sum_{\substack{j \neq i \\ j \in I}} \delta^{\eta t_{ij}}$$

Noting that the values of x_i^* do not depend on the affiliation decisions of agents other than i,[7] it is easy to see that the previous expression has the following structure:

$$\alpha_i = \begin{cases} 0 & \text{if } \quad b_i < 0 \\ 0 \text{ or } 1 & \text{if } \quad b_i = 0 \\ 1 & \text{if } \quad b_i > 0 \end{cases} \tag{10}$$

where $b_i \in \mathbb{R}$ with:

$$b_i = x_i - \frac{1}{2} - \mathbf{1}^t e_{-i}(p_A, p_B)$$

where:

[7] In the following section we study the discriminatory case, where x_i^* actually depends on the affiliation decisions of all the agents, and the problem becomes much more complicated.

$$e_{-i}(p,q) = \begin{pmatrix} e_{i,1}(p,q) \\ \vdots \\ e_{i,i-1}(p,q) \\ e_{i,i+1}(p,q) \\ \vdots \\ e_{i,I}(p,q) \end{pmatrix}_{I-1} \qquad \mathbf{1} = \begin{pmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix}_{I-1}$$

$$e_{i,j}(p,q) = \frac{\sigma_i \delta^{\eta t_{ij}}}{\eta - 1} \left(p^{1-\eta} - q^{1-\eta}\right)$$

The optimal affiliation decisions are then formally characterized, but they are still nonlinear. In order to linearize them, consider $M \in \mathbb{R}_+$ sufficiently high such that, for given i, constraint (10) is equivalent to the following couple of inequations:[8]

$$\begin{aligned} 0 &\geq b_i - M\alpha_i \\ 0 &\leq b_i + M(1-\alpha_i) \end{aligned} \tag{11}$$

In effect, when $b_i < 0$ holds, agent i is forced to choose $\alpha_i = 0$ otherwise (i.e. by selecting $\alpha_i = 1$) the second inequality in (11) is violated. An analogous argument applies when $b_i > 0$. In the case when $b_i = 0$ the inequalities in (11) hold with $\alpha_i = 0$ or $\alpha_i = 1$. As a result, the vector of affiliation decisions must satisfy the following system of linear inequations:

$$H_1 \alpha \leq z_1$$

where:

[8] A feasible definition of M is given in Appendix I.

$$H_1 = \begin{bmatrix} -M & & & \\ M & & & \\ & -M & & \\ & M & & \\ \vdots & \vdots & \vdots & \\ & & -M & \\ & & M & \end{bmatrix}_{2I\times I} \qquad z_1 = \begin{bmatrix} -b_1 \\ b_1 + M \\ -b_2 \\ b_2 + M \\ \vdots \\ -b_I \\ b_I + M \end{bmatrix}_{2I\times 1} \qquad \alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_I \end{pmatrix}_{I\times 1}$$

It is convenient to emphasize that H_1 is independent of a particular vector of prices (p_A, p_B). On the other hand, z depends on the vector of prices because b_i does so for each i. Accordingly we should write the constraint as: $H_1\alpha \leq z_1(p_A, p_B)$.

3.1.2 The Objective Function

Consider the problem for firm A, established in equation (7), under the relevant constraints. By replacing the optimal values for q_{ij} defined in equation (3) it becomes:

$$\begin{aligned}\max_{p_A\geq 0}\pi_A(p_A, p_B, a_A, a_B) = (p_A - c_A^o - c_A^f)p_A^{-\eta}\sum_{i\in I_A}\sum_{\substack{j\neq i\\ j\in I_A}}\delta^{\eta t_{ij}} + (p_A - c_A^o - a_B)p_A^{-\eta}\sum_{i\in I_A}\sum_{j\in I_B}\delta^{\eta t_{ij}} \\ - \sum_{i\in I_A} f + (a_A - c_A^f)p_B^{-\eta}\sum_{i\in I_B}\sum_{j\in I_A}\delta^{\eta t_{ij}}\end{aligned}$$

It is important to remember that the previous structure of the objective function was inadequate because the sets I_A and I_B represent the group of consumers affiliated to the corresponding firms, which are endogenous to the vector of prices (p_A, p_B). The objective function can be simplified by incorporating the variables α_i identifying the affiliation decisions. If we include the fact that affiliation decisions are also optimal for consumers, we have that firm A's problem is given by:

$$\max_{p_A \geq 0} \pi_A(p_A, p_B, a_A, a_B; \alpha) = (p_A - c_A^o - c_A^f) p_A^{-\eta} \sum_{i \in I} \sum_{\substack{j \neq i \\ j \in I}} \delta^{\eta t_{ij}} (1 - \alpha_i)(1 - \alpha_j) \tag{12}$$

$$+ (p_A - c_A^o - a_B) p_A^{-\eta} \sum_{i \in I} \sum_{j \in I} \delta^{\eta t_{ij}} (1 - \alpha_i) \alpha_j$$

$$- \sum_{i \in I} (1 - \alpha_i) f + (a_A - c_A^f) p_B^{-\eta} \sum_{i \in I} \sum_{j \in I} \delta^{\eta t_{ij}} \alpha_i (1 - \alpha_j)$$

$$s.t. \qquad H_1 \alpha \leq z_1(p_A, p_B), \qquad \alpha \in \{0, 1\}^I$$

where H, z and α were defined in the previous subsection. It is clear that problem (12) has the structure required in (8).

The analogous problem for Firm B is trivially given by:

$$\max_{p_B \geq 0} \pi_B(p_A, p_B, a_A, a_B; \alpha) = (p_B - c_B^o - c_B^f) p_B^{-\eta} \sum_{i \in I} \sum_{\substack{j \neq i \\ j \in I}} \delta^{\eta t_{ij}} \alpha_i \alpha_j \tag{13}$$

$$+ (p_B - c_B^o - a_A) p_B^{-\eta} \sum_{i \in I} \sum_{j \in I} \delta^{\eta t_{ij}} \alpha_i (1 - \alpha_j)$$

$$- \sum_{i \in I} \alpha_i f + (a_B - c_B^f) p_A^{-\eta} \sum_{i \in I} \sum_{j \in I} \delta^{\eta t_{ij}} (1 - \alpha_i) \alpha_j$$

$$s.t. \qquad H_1 \alpha \leq z_1(p_A, p_B), \qquad \alpha \in \{0, 1\}^I$$

Note that the constraint is the same that in equation (12), even when the objective function changes according to the definition of α_i's.

3.2 Case 2.a: Discriminating by Destiny (on-off net price discrimination)

As before, we assume that for all the consumers the individual rationality constraint is not binding, i.e., $\beta_i = 1 \; \forall i \in I$. In the same way as in the linear case, adding a constant to the utility function is enough to satisfy this condition. It is easy to see that α_i represents affiliation decisions and we can write:

$$\alpha_i = \begin{cases} 0 & \text{if} \quad b_i < L_i^t \alpha_{-i} \\ 0 \text{ or } 1 \text{ if} & b_i = L_i^t \alpha_{-i} \\ 1 & \text{if} \quad b_i > L_i^t \alpha_{-i} \end{cases} \tag{14}$$

where α_{-i} is a $I-1$ column vector containing the affiliation decisions of agents other than i, L_i is a $I-1$ column vector and $b_i \in \mathbb{R}$ with:

$$b_i = x_i - \frac{1}{2} - \mathbf{1}^t e_{-i}(p_A, \widehat{p}_B)$$
$$L_i = e_{-i}(\widehat{p}_A, p_B) - e_{-i}(p_A, \widehat{p}_B)$$

where:

$$e_{-i}(p,q) = \begin{pmatrix} e_{i,1}(p,q) \\ \vdots \\ e_{i,i-1}(p,q) \\ e_{i,i+1}(p,q) \\ \vdots \\ e_{i,I}(p,q) \end{pmatrix}_{I-1} \qquad \alpha_{-i} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_{i-1} \\ \alpha_{i+1} \\ \vdots \\ \alpha_I \end{pmatrix}_{I-1} \qquad \mathbf{1} = \begin{pmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix}_{I-1}$$

$$e_{i,j}(p,q) = \frac{\sigma_i \delta^{\eta t_{ij}}}{\eta - 1} \left(p^{1-\eta} - q^{1-\eta}\right)$$

Note that different from equation (10) the condition in equation (14) depends on where *ís* contacts are affiliated. In particular if $x_i > x_j$ and individual i affiliates to firm A, it does not implies that individual j would prefer firm A. The reason behind is that *j́s* social contacts could be affiliated to B, so j could prefer B to take advantage of more convenient on net prices. This result shows that a consumption inefficiency arises as a consecuence of on-off net price discrimination.

The constraint (14) is still hard to incorporate in an optimization program. We would like to have a linearized version of this constraint which should be impossed $\forall i \in I$.

Consider M sufficiently high[9] such that, for given i, the expression (14) is equivalent

[9] A feasible definition of M is given in the Appendix 1.

to the following couple of inequations:

$$L_i^t \alpha_{-i} \geq b_i - M\alpha_i \qquad (15)$$
$$L_i^t \alpha_{-i} \leq b_i + M(1-\alpha_i)$$

In effect, when $b_i < L_i^t \alpha_{-i}$ holds, agent i is forced to choose $\alpha_i = 0$ otherwise (i.e. by selecting $\alpha_i = 1$) the second inequality in (15) is violated. An analogous argument applies when $b_i > L_i^t \alpha_{-i}$. In the case when $b_i = L_i^t \alpha_{-i}$ the inequalities in (15) are satisfied with either $\alpha_i = 0$ or $\alpha_i = 1$.

As a result, the vector of affiliation decisions must satisfy the following system of linear inequations:

$$H_1^I \alpha \leq z_1^I$$

where:

$$H_1^I = \begin{bmatrix} -M & -(L_1^D)^t & \\ M & (L_1^D)^t & \\ -(L_2^U)^t & -M & -(L_2^D)^t \\ (L_2^U)^t & M & (L_2^D)^t \\ \vdots & \vdots & \vdots \\ -(L_i^U)^t & -M & -(L_i^D)^t \\ (L_i^U)^t & M & (L_i^D)^t \\ \vdots & \vdots & \vdots \\ & -(L_I^U)^t & -M \\ & (L_I^U)^t & M \end{bmatrix}_{2I \times I} \qquad z_1^I = \begin{bmatrix} -b_1 \\ b_1 + M \\ -b_2 \\ b_2 + M \\ \vdots \\ \vdots \\ -b_I \\ b_I + M \end{bmatrix}_{2I \times 1} \qquad \alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_I \end{pmatrix}_{I \times 1}$$

and:

$$L_i = \begin{pmatrix} L_{i,1} \\ \vdots \\ L_{i,i-1} \\ L_{i,i+1} \\ \vdots \\ L_{i,I} \end{pmatrix} \equiv \begin{pmatrix} L_i^U \\ L_i^D \end{pmatrix} \qquad \alpha_{-i} \equiv \begin{pmatrix} \alpha_{-i}^U \\ \alpha_{-i}^D \end{pmatrix} \qquad \alpha = \begin{pmatrix} \alpha_{-i}^U \\ \alpha_i \\ \alpha_{-i}^D \end{pmatrix}$$

It is convenient to emphasize that H_1^I depends on the vector of prices $(p_A, p_B, \widehat{p}_A, \widehat{p}_B)$ because L_i does for each i. Analogously, z_1^I also depends on the vector of prices because b_i does for each i.

Consider now the objective function for firm A, established in equation (7), but in the absense of fixed charges:

$$\pi_A^I(p_A, \widehat{p}_A, p_B, \widehat{p}_B, a_A, a_B) = (p_A - c_A^o - c_A^f)p_A^{-\eta} \sum_{i \in I_A} \sum_{\substack{j \neq i \\ j \in I_A}} \delta^{\eta t_{ij}} + (\widehat{p}_A - c_A^o - a_B)\widehat{p}_A^{-\eta} \sum_{i \in I_A} \sum_{j \in I_B} \delta^{\eta t_{ij}}$$
$$- \sum_{i \in I_A} f + (a_A - c_A^f)\widehat{p}_B^{-\eta} \sum_{i \in I_B} \sum_{j \in I_A} \delta^{\eta t_{ij}}$$

and using the definition of α_i we have:

$$\begin{aligned} \pi_A^I(p_A, \widehat{p}_A, p_B, \widehat{p}_B, a_A, a_B; \alpha) = {} & (p_A - c_A^o - c_A^f)p_A^{-\eta} \sum_{i \in I} \sum_{\substack{j \neq i \\ j \in I}} \delta^{\eta t_{ij}} (1 - \alpha_i)(1 - \alpha_j) \\ & + (\widehat{p}_A - c_A^o - a_B)\widehat{p}_A^{-\eta} \sum_{i \in I} \sum_{j \in I} \delta^{\eta t_{ij}} (1 - \alpha_i)\alpha_j \\ & - \sum_{i \in I} (1 - \alpha_i) f + (a_A - c_A^f)\widehat{p}_B^{-\eta} \sum_{i \in I} \sum_{j \in I} \delta^{\eta t_{ij}} \alpha_i (1 - \alpha_j) \end{aligned}$$

Accordingly, the problem for firm A has been transformed into:

$$\underset{p_A, \widehat{p}_A \geq 0}{Max} \; \pi_A^I(p_A, \widehat{p}_A, p_B, \widehat{p}_B, a_A, a_B; \alpha) \tag{16}$$

$$s.t. \quad H_1^I \alpha \leq z_1^I, \quad \alpha \in \{0,1\}^I \qquad (IC \text{ constraints})$$

It is important to note that the linear non discriminatory case (case 1) can be obtained as a special case of this problem (where $p_A = \widehat{p}_A$ and $p_B = \widehat{p}_B$).

3.3 Case 2.b: Two part Tariffs

It is easy to see that the incentive compatibility constraint is a particular case of the analysis in the previous subsection with $p_A = \widehat{p}_A$ and $p_B = \widehat{p}_B$. In such a case $L_i = 0$ $\forall i \in I$ so matrix H_1^{II} is considerably simpler. Additionaly the expression for b_i would naturally become $b_i = x_i - \frac{1}{2} - \mathbf{1}^t e_{-i}(p_A, p_B) - \sigma_i(F_B - F_A)$. The interpretation for α_i, however, is back to one of preferences instead of affiliation decisions.

An analogous procedure let us to establish an N sufficiently high such that equation (6) is equivalent to:

$$\begin{aligned} 0 &\geq \Omega_i - N\beta_i \\ 0 &\leq \Omega_i - N(1-\beta_i) \end{aligned} \tag{17}$$

As a result, the vector of market participation decisions must satisfy the following system of linear inequations:

$$H_2^{II} \beta \leq z_2^{II}$$

where:

$$H_2^{II} = \begin{bmatrix} -N & & \\ N & & \\ & -N & \\ & N & \\ \vdots & \vdots & \vdots \\ & & -N \\ & & N \end{bmatrix}_{2I \times I} \qquad z_2^{II} = \begin{bmatrix} -\Omega_1 \\ \Omega_1 + N \\ -\Omega_2 \\ \Omega_2 + N \\ \vdots \\ -\Omega_I \\ \Omega_I + N \end{bmatrix}_{2I \times 1} \qquad \beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_I \end{pmatrix}_{I \times 1}$$

In this case, it is convenient to emphasize that H_2^{II} is independent of a particular vector of prices (p_A, p_B). However, z_2^{II} depends now on the vector of prices, fixed charges and also on α_i because Ω_i does for each i.

Consider now the objective function for firm A, established in equation (7), but in the absense of destiny based discrimination:

$$\begin{aligned}
&\pi_A^{II}(p_A, p_B, F_A, F_B, a_A, a_B) = \\
&\textstyle\sum_{i\in I_A} \left\{ \sum_{\substack{j\in I\setminus I_B \\ j\neq i}} q_{ij}(p_A)(p_A - c_A^o - c_A^f) + \sum_{j\in I_B} q_{ij}(p_A)(p_A - c_A^o - a_B) + F_A - f \right\} + \\
&\textstyle\sum_{i\in I_B} \sum_{j\in I_A} q_{ij}(p_B)(a_A - c_A^f)
\end{aligned}$$

Assuming symmetric access charges it becomes:[10]

$$\begin{aligned}
&\pi_A^{II}(p_A, p_B, F_A, F_B, a, a) = \\
&\textstyle\sum_{i\in I_A} \left\{ \sum_{\substack{j\in I \\ j\neq i}} q_{ij}(p_A)(p_A - c_A^o - c_A^f) + F_A - f \right\} + \\
&\textstyle\sum_{i\in I_A} \sum_{j\in I_B} \left(q_{ji}(p_B) - q_{ij}(p_A)\right)(a - c_A^f)
\end{aligned}$$

and using the definitions of α_i and β_i we have:

$$\begin{aligned}
&\pi_A^{II}(p_A, p_B, F_A, F_B, a, a; \alpha, \beta) = \\
&\textstyle\sum_{i\in I}(1-\alpha_i)\beta_i \left\{ \sum_{\substack{j\in I \\ j\neq i}} q_{ij}(p_A)(p_A - c_A^o - c_A^f) + F_A - f \right\} + \\
&\textstyle\sum_{i\in I}(1-\alpha_i)\beta_i \sum_{\substack{j\in I \\ j\neq i}} \alpha_j\beta_j \left(q_{ji}(p_B) - q_{ij}(p_A)\right)(a - c_A^f)
\end{aligned}$$

Accordingly, the problem for firm A has been transformed into:

$$\underset{p_A, F_A \geq 0}{Max}\ \pi_A^{II}(p_A, p_B, F_A, F_B, a, a; \alpha, \beta) \tag{18}$$

[10] This assumption is not essential for the model, but it is an standard assumption that simplifies our expressions. In fact it is an usual legal constraint in countries where a is regulated, as well as in countries where the operators can agree on a mutual access charge. The only exception occurs when the regulatory authority wants to impose an asimmetry to correct other distortion. For example, when the incumbent has market power and the authority wants to foster competition from new entrants. In such a case, assymetric access charges can be defined by the regulator.

$$
\begin{aligned}
s.t. \quad & H_1^{II}\alpha \le z_1^{II}, \quad \alpha \in \{0,1\}^I \qquad (IC \text{ constraints}) \\
& H_2^{II}\beta \le z_2^{II}, \quad \beta \in \{0,1\}^I \qquad (IR \text{ constraints})
\end{aligned}
$$

It is interesting to note here that if we remove the assumption that disconnected individuals can receive calls, then some terms can appear out of the bidiagonal in matrix H_2^{II}. The analogy with the difference between H_1 and H_1^I is evident. In that case, the difference is given by the presence of *tariff mediated network externalities,* which makes the selection of a provider an interdependent decision. In the present case, when disconnected individuals are not able to receive calls, then market participation decisions become interdependent, because if some individual decides to remain our of the market, the utility for other individuals would diminish, specially those socially connected. As a result, a *participation network externality* arises.

4 Example of Application: Regulatory Interventions

In the following analysis we consider a regulatory application of our model. In particular, we consider the standard regulatory approach, where access charges are defined by the authority, and only the final prices are the result of market interactions. In the benchmark cases, the authority selects access charges as equal to marginal termination costs (i.e. $a_A = c_A^f$ and $a_B = c_B^f$). For simplicity we also assume symmetric firms so that $c_A^f = c_B^f$. Departing from this benchmark, we have two alternative regulatory interventions:

1. The authority can set access charges below marginal termination costs to enhance competition. Under this policy, the firms have an additional incentive to reduce prices, because a net outflow of calls is more profitable than a balanced pattern.

2. The authority can implement policies equivalent to reducing transportation costs, that is τ, which intensify rivalry to affiliate consumers.

We are going to describe how equilibrium is affected under each regulatory intervention and then we will perform a comparative analysis for the welfare achieved in both of them and in relation to reference cases.

The welfare analysis can be constrained to a simple comparison between the results of both regulatory interventions, but it is also illustrative to compare those results with some relevant benchmarks. A first benchmark considered is given by the standard access regulation described above, but final prices are set monopolically. A second one is a

Ramsey approach, where consumer surplus is maximized subject to an industry break even constraint. In this section we provide the fundamental tools to set the benchmarks and to discuss regulatory interventions.

For any pair of prices (p_A, p_B) we can evaluate consumer surplus as:

$$CS(p_A, \widehat{p}_A, p_B, \widehat{p}_B, F_A, F_B) = \sum_{i \in I_A} w_i(p_A, \widehat{p}_A, F_A, x_i) + \sum_{i \in I_B} w_i(p_B, \widehat{p}_B, F_B, 1 - x_i) \quad (19)$$

Accordingly, total welfare could be defined by:

$$\begin{aligned} TW(p_A, p_B) &= CS(p_A, p_A, p_B, p_B, F_A, F_B) + \pi_A(p_A, p_A, p_B, p_B, F_A, F_B, c_A^f, c_B^f) \\ &\quad + \pi_B(p_A, p_A, p_B, p_B, F_A, F_B, c_A^f, c_B^f) \end{aligned}$$

It is worthy to note that in the obtention of total welfare we are not considering on-off price discrimination and that F_A and F_B are just transfers from consumers to firms, so they should not affect welfare as long as all the consumers are attended. We then could evaluate how close is welfare obtained in equilibrium from the maximum achievable welfare given by:

$$\underset{p_A,\ p_B}{Max}\ TW(p_A, p_B)$$

Unfortunately, consumer surplus can not be directly added to profits, because the multiple ways to consider transportation costs in an horizontally differentiated model implies multiple measures for consumer surplus. An alternative approach that permit us to avoid this problem is the second best solution associated to the Ramsey problem:[11]

$$\begin{aligned} &\underset{p_A,\ p_B \geq 0}{Max}\ CS(p_A, p_A, p_B, p_B, 0, 0) \qquad (20) \\ &s.t. \\ &\pi_A(p_A, p_A, p_B, p_B, 0, 0, c_A^f, c_B^f) + \pi_B(p_A, p_A, p_B, p_B, 0, 0, c_A^f, c_B^f) = 0 \end{aligned}$$

[11] The same approach for the Ramsey option is followed by Laffont *et al.* (1998a), where the problem does not contain a social structure among customers.

Where access charges have been set as equal to marginal termination costs. In this approach we can compare the Ramsey solution, given by equation (20), with the values obtained in equilibrium under alternative regulatory interventions.

Finally, it is also illustrative to use the monopoly case as another benchmark. In this case the affiliation decision is irrelevant and the firm simply solves:

$$\max_{p\geq 0} \pi_M(p) = \left\{ (p - c_M^o - c_M^f) p^{-\eta} \sum_{i\in I} \sum_{\substack{j\neq i \\ j\in I}} \delta^{\eta t_{ij}} - \sum_{i\in I} f \right\}$$

Where the subindex M denotes monopoly levels.

4.1 Simulation Results

In this section we report the main simulation results for alternative regulatory interventions. It should be noted that problems (16) and (18) are nonlinear not only in the objective function, but also in the constraints, because they depend on prices. For any given vector of prices the constraint can be solved in α and/or β. Once α and/or β has been selected, we can evaluate the goal function for the corresponding vector of prices, access charges, α and β. We look for a symmetric Nash Equilibrium in all the settings.

The default values for the parameters are given in Table 1. In the subsequent analysis below, we depart from this setting in some key variables associated to different regulatory interventions.

Table 1: Default Values for Parameters

elasticity of demand	$-\eta = -1.2$
discount factor	$\delta = 0.9$
origination cost	$c_A^o = c_B^o = 0.75$
termination cost	$c_A^f = c_B^f = 0.75$
fix cost	$f = 50$
access charges	$a_A = a_B = 0.75$
number of individuals	$I = \begin{cases} 1000 \text{ in case } 1 \\ 100 \text{ in cases } 2a \text{ and } 2b \end{cases}$
transportation cost	$\tau = \begin{cases} 0.5 \text{ in case } 2a \\ 0.25 \text{ in cases } 1 \text{ and } 2b \end{cases}$

All the numbers in Table 1 were selected trying to conform a reasonable setting. For example, Ingraham and Sidak (2004) have estimated that the elasticity of demand in US for wireless services is between -1.12 and -1.29. The fixed cost (f) has been selected in order to represent 10% of ARPU (Average Revenue per User). On the other hand, origination, termination and transportation costs are in the same order of magnitude reported by De Bijl and Peitz (2002) in their simulations.

For the analysis of the regulatory interventions, Table 2 summarize the setting for the parameters. The first column corresponds to the standard case where regulatory authorities set access prices as equal to marginal termination costs. The second column contains the parameters for the Ramsey approach, while the last two columns contain the settings where the intervention occurs in access charges (scheme 1) and in transportation costs (scheme 2), respectively.

Table 2: Basic Parameters under Regulatory Interventions

Parameters	Standard	Ramsey	Scheme 1	Scheme 2
access charges ($a_A = a_B$)	0.75	0.75	*variable*	0.75
transportation cost (τ)	as in Table 1	as in Table 1	as in Table 1	*variable*
connectivity degree (d)	*variable*	*variable*	*variable*	*variable*

4.2 Results for Case 1: Linear Non Discriminatory Prices

For Figures 1 through 5 we report average results over 15 random networks generated for each average level of connectivity degree (d).

Figure 1 shows how the connectivity degree d affects average equilibrium prices as well as Ramsey prices. The main impact is observed at low levels of connectivity. However, Figure 2 shows that connectivity degree is an important factor affecting consumer surplus for all d. Although the authority can not read this result as implying a policy intervention to increase d, it is clear that the gap between the equilibrium and the Ramsey benchmark can be reduced through regulation, and this is especially relevant for high values of d. On the other hand, the gap between the equilibrium and the monopoly case increases in d, showing that the importance of pro competitive regulation increases when individuals become more socially connected. Finally, Figure 3 shows the gap between total profits under competition versus monopoly, both of them consider the standard regulatory environment. It is clear that the industry payoff from colusion is higher when the connectivity degree increases.

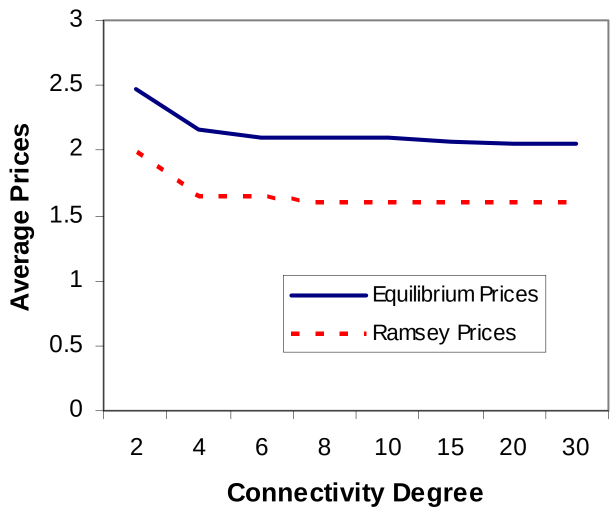

Figure 1: The impact of Connectivity degree (d) on Equilibrium prices.

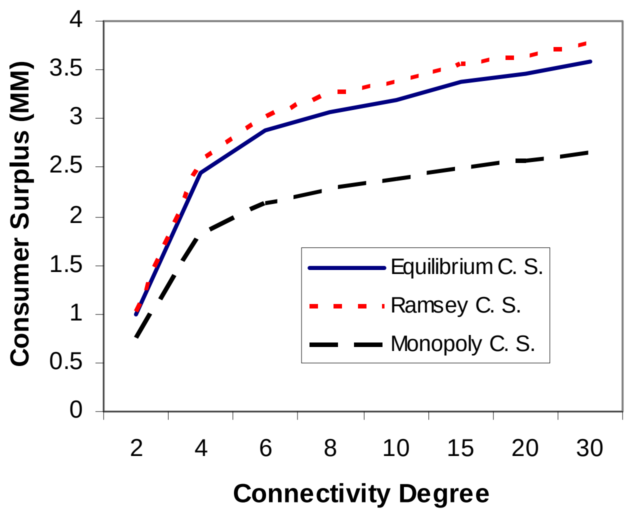

Figure 2: The impact of Connectivity degree (d) on Consumer Surplus (CS).

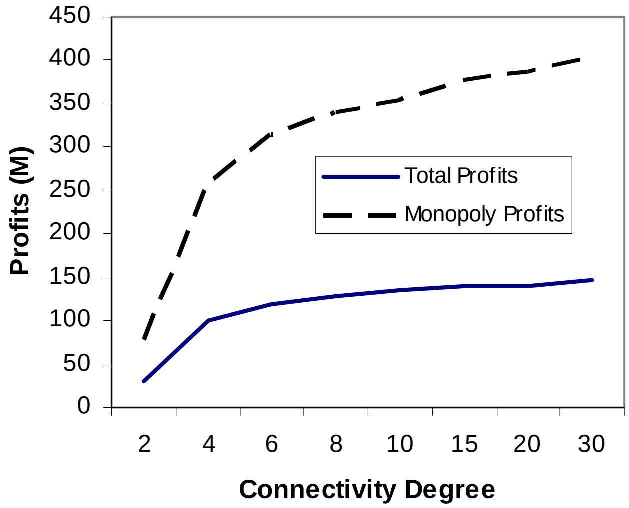

Figure 3: The impact of Connectivity degree (d) on Producer Surplus.

In order to study the effect of the random network-generating process over the results, we slightly modified the way how the network is obtained. Instead of controlling just the probability of connection to meet the desired connectivity degree, we also controlled the variance of that probability (v). When $var = 0$ we have the original case when the probability of connection is the same accross individuals, but $var = 0.25$ means that this probability can result 25% higher or lower than the goal value. Figure 4 shows that a significant effect appears only for very low connectivity degrees.

We already mentioned that competition in retail markets increases when access charges are settled below marginal costs. Figure 5 provides support for this policy recomendation, showing that equilibrium prices can get closer to Ramsey levels when access charges are reduced. It is clear, according to our simulations, that lowering access charges even below marginal termination costs permits us to increase social welfare. It is also clear that the higher the connectivity degree, the lower the equilibrium prices reached,[12] and then the most effective the policy. However, this is not the only policy intervention that can be evaluated. Figure 6 shows the effect of a policy where the authority reduces transportation costs, intensifying the competition for customers. Although it is difficult to find a direct policy reducing transportation costs, it is easy to find policies leading to equivalent effects

[12] The average rate of reduction over connectivity degrees was 1.95%.

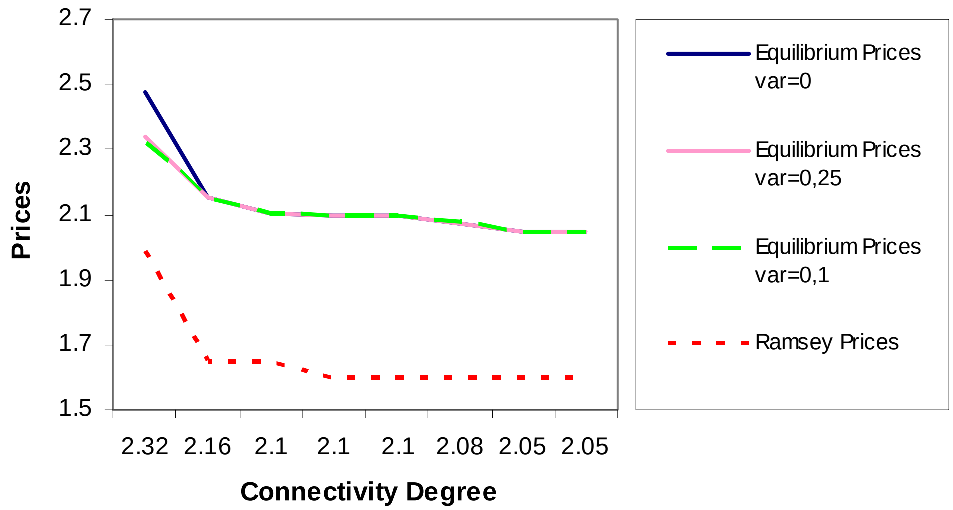

Figure 4: Price Sensitivity to the Graph Generation Process.

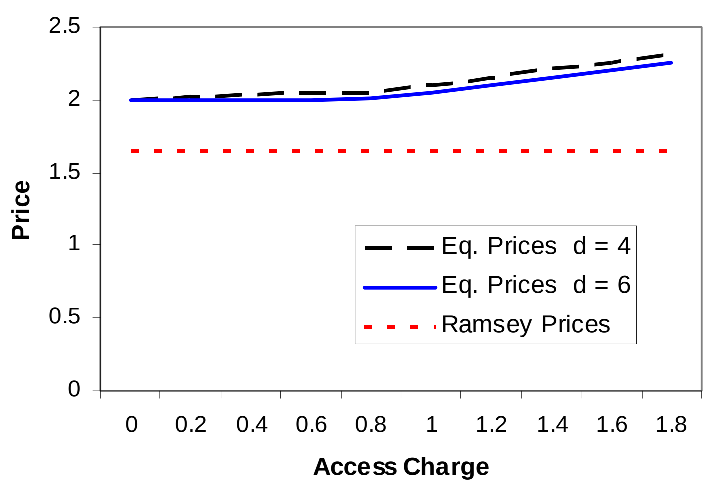

Figure 5: The Effect of Access Charges on the gap between Equilibrium and Ramsey Prices.

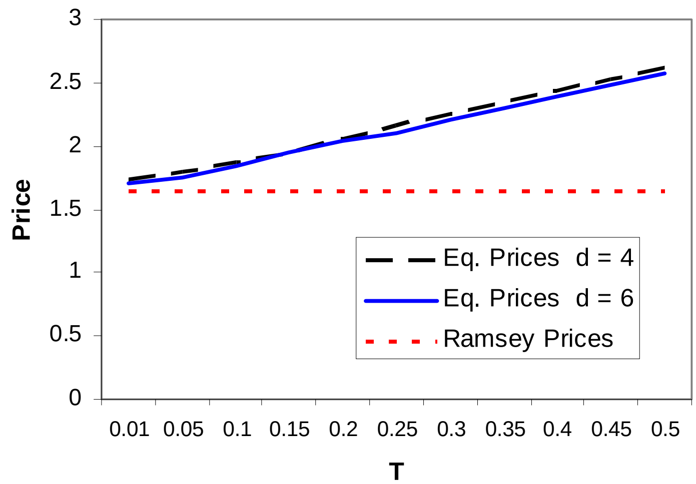

Figure 6: The Effect of reducing Transportation Costs on the gap between Equilibrium and Ramsey Prices

on equilibrium prices. For example, a policy aimed to reduce the differentiation of services can increase competition leading to the same equilibrium prices as a given reduction in transportation costs.[13][14] Another option is a reduction in switching costs, for example, in the static framework of this paper, one of such policy interventions would be the implementation of number portability.[15] Our simulations show that this kind of policies are even more effective than access charge regulation in generating equilibrium prices closer to the Ramsey benchmark case. As before, the effectiveness of the policy increases the higher the connectivity degree is, however the rate of reduction in prices is lower than in the access charge policy approach,[16] probably because the benchmark situation (with $d = 4$) is already close to Ramsey prices.

4.3 Results for Case 2.a: Discrimination by Destiny

In this case the structure of matrix H_1^I is complex enough for the constraint to admit multiple equilibria. The criteria here was to select α so as to minimize $\sum_{i=1}^{I} \alpha_i$. In other

[13] The effect on welfare is different, because total transportation costs are reduced when differentiation is limited. In other words, this policy can be socially superior to a "price equivalent" reduction in transportation costs.

[14] Policies oriented to set quality standards, coverage areas, etc.

[15] Transportation costs can be interpreted as switching costs in the following way. Customers can be considered as assigned to the "closer" firm, but if they want to buy the service from the other, they can do it but paying a switching cost equal to the difference in transportation costs.

[16] In this case the rate of reduction was 1.63%.

words, the most favorable selection fo firm A in terms of market share.

Main results are summarized in figures 7 to 9. In order to be able to compare with reference cases, we also provide the results for the Ramsey and the monopoly case. In Figure 7 we show the dependance of consumer surplus on the connectivity degree (d). It is clear from the figure that social structure matters.

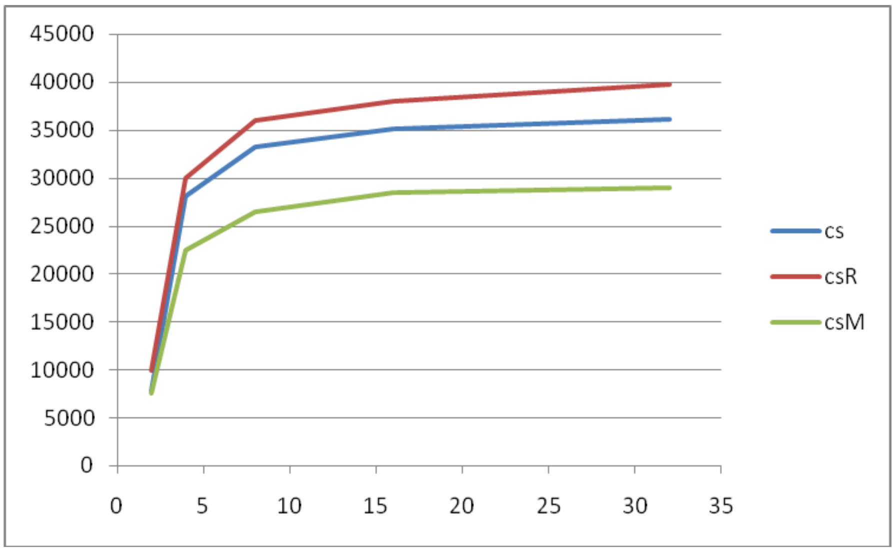

Figure 7: The impact of Connectivity degree on Consumer Surplus (CS).

Figure 8 reports the equilibrium results for both average prices (p and $\widehat{p}$) when access charges are permitted to change. Both of them are above the Ramsey prices and by far below monopoly prices, but the most interesting finding is that for sufficiently low access charges it is cheaper to call off net than on net. The reason is simple, receiving calls from the rival firm is expensive, because the termination cost is higher than the access charge. As a result, firms try to attract high demand customers to avoid a high flow coming from the rival network.

Figure 9 reports the equilibrium results for both average prices (p and $\widehat{p}$) when transportation costs are permitted to change. Both of them are above the Ramsey prices, but they are closer than in Figure 8. It is interesting that for low transportation costs it is cheaper to call off net than on net. The reason for this behavior, however, is quite different from the case of access charge regulation described in Figure 8. If in equilibrium $\widehat{p}$ were higher than p then all the consumers would affiliate one network and, in the static framework under analysis, the competition would be intensified because the firm that loses the

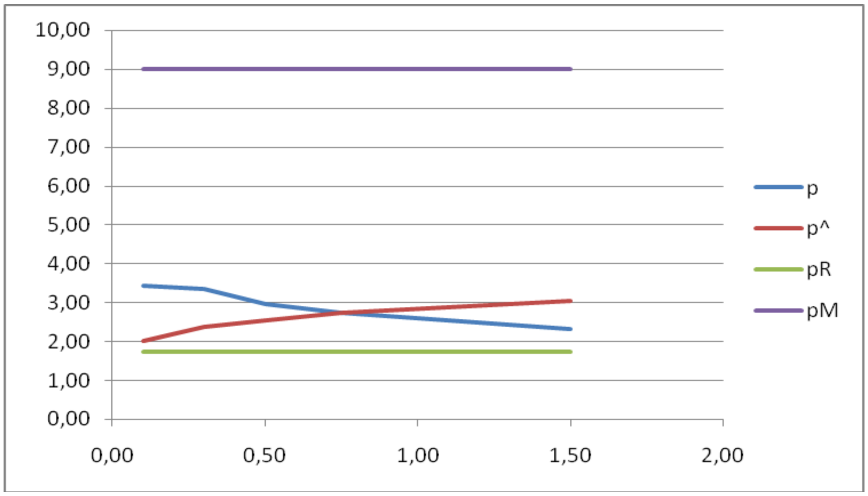

Figure 8: Average equilibrium prices (p and $\widehat{p}$) as a function of access charges.

battle is out of the market. Moreover, given the rule to select among multiple equilibria, the surviving firm would be A.

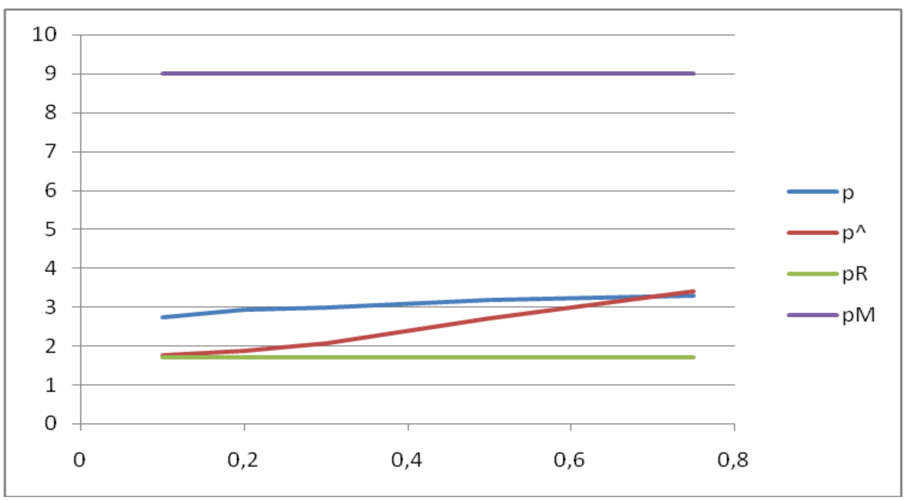

Figure 9: Average equilibrium prices (p and $\widehat{p}$) as a function of transportation costs.

4.4 Results for Case 2.b: Two part Tariffs

The case of nonlinear pricing schemes is different in several aspects. First, in this case an important issue is the non trivial participation decision in the market ($\beta_i = 1$ versus 0).

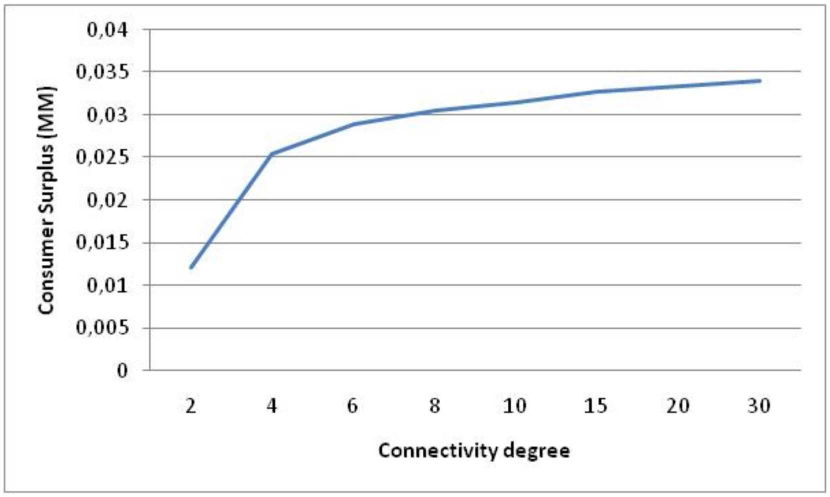

Figure 10: The impact of connectivity degree on Consumer Surplus (CS).

Second, uniqueness in the solution for affiliation decisions is guaranteed. This is because both matrix H_1^{II} and H_2^{II} are "diagonal". The net effect is that new algorithms are simpler than those developed for case 2.a.

Some results are summarized in figures 10 to 12. In Figure 10 we show the dependence of consumer surplus on the connectivity degree (d). As in the previous case, it is clear from the figure that social structure matters.

In order to study the relative efficiency of the two kinds of regulations: access charges v/s transportation costs, it is convenient to focus the analysis on the effect of each variable on Consumer Surplus. Figures 11 and 12 show that reductions in transportation costs have a positive and predictable effect on Consumer Surplus, while access charge control is not longer useful as a tool to increase consumer surplus in the presence of two part tariffs. The intuition is that a reduction in access charges can be accompanied by an increase in the fixed part of a two part tariff and in equilibrium, some customers can be excluded from the market.[17]

5 Conclusion

In this paper we provide a model to study the competition between two interconnected firms offering differentiated communication services in a context where consumers are re-

[17] This negative effect would be incremented if the excluded customer were not reachable by the others in our model.

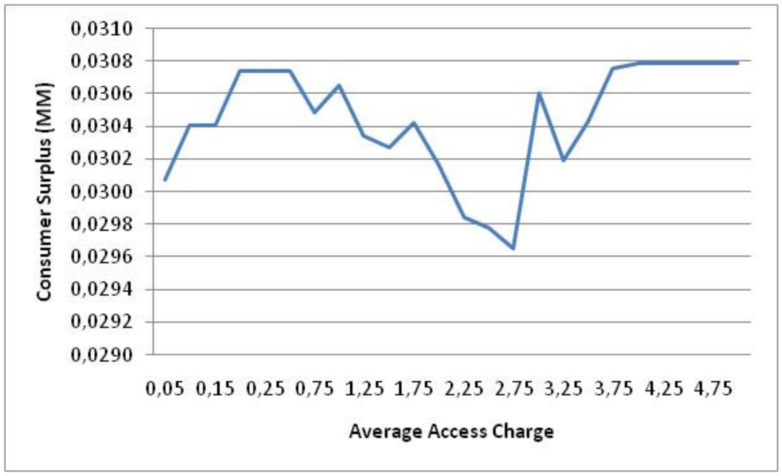

Figure 11: Consumer Surplus as a Function of Average Access Charge.

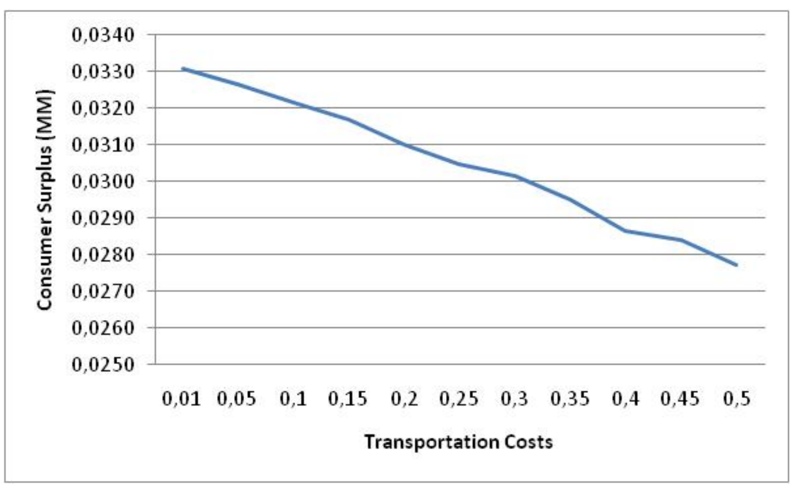

Figure 12: Consumer Surplus as a Function of Transportation Costs.

lated through a social network. Our goal was to build a sufficiently flexible model to study different structures related to the regulatory environment and pricing schemes permitted. The main difference with the existing literature is that the analysis was performed using a model where rational consumers are related through a social network, and then the number of calls (or direct conections) between any pair of them depends not only on prices and transportation costs, but also on how socially close they are in the network.

The results showed that equilibrium prices, consumer surplus and producer surplus depend on the connectivity parameter d, showing that social networks matter in the way how markets perform and also how regulation should be accomplished. For example, although the regulatory role of the authority seems to be mandatory, its importance depends on the social network characteristics, because the collusive scenario, associated to monopoly outcomes, is more profitable for the coalition, and then has higher impact on consumer surplus, the higher the connectivity degree is in the social network. On the other hand, we explore the impact of introduce some perturbations in the random network-generating process, keeping constant the connectivity degree. For the perturbations considered, the impact on results was of second order.

In relation to the impact of regulatory interventions, our results showed that under linear tariffs setting access charges below marginal costs have a positive impact on competition, reducing equilibrium prices to consumers. However, alternative policy interventions, represented by reductions in transportation costs, were much more effective at any connectivity degree, because they brought final prices closer to a second best solution given by the Ramsey approach. Under nonlinear pricing schemes other effects arise. For example, when discrimination by destiny is permitted, setting access charges below marginal costs implies that in equilibrium off net calls would become cheaper than on net calls. On the other hand, when two part tariffs are feasible, consumer surplus is effectively increased by a policy oriented to reduce transportation costs, but a policy focused on reducing access charges is not longer useful. In this line, policies such as number portability appear as highly desirable in telecommunication markets in front of policies oriented just to bound interconection charges in calling party pays regimes.

6 Appendix 1: Definition of M and N

The goal of this section is to define valid values for the bounds M and N introduced in equations (15) and (17), respectively.

In the case of M we consider:

$$|b_i - L_i^t \alpha_{-i}| \leq |b_i| + |L_i^t \alpha_{-i}| \leq \tfrac{1}{2} + \tfrac{\sigma_i}{\eta-1} \left(\underline{p}^{1-\eta} - \overline{p}^{1-\eta}\right) \sum_{\substack{j\in I \\ j\neq i}} \delta^{\eta t_{ij}} + 2\tfrac{\sigma_i}{\eta-1} \left(\underline{p}^{1-\eta} - \overline{p}^{1-\eta}\right) \sum_{\substack{j\in I \\ j\neq i}} \delta^{\eta t_{ij}}$$

using that $0 < \delta < 1$ and $\eta > 1$ we have:

$\leq \frac{1}{2} + \frac{3\sigma_i}{\eta-1}(I-1)\left[\underline{p}^{1-\eta} - \overline{p}^{1-\eta}\right]$

where underbar and upperbar represent the minimum and maximum possible value for the corresponding variable.

Assuming that individual i is connected to the network $\left(\sum_{j\neq i} \delta^{t_{ij}} \geq 1\right)$ we have:[18]

$\sigma_i \leq \sigma \equiv \frac{1}{2\tau}$ and then M can be chosen as:

$M \equiv \frac{1}{2} + \frac{3\sigma}{\eta-1}(I-1)\left[\underline{p}^{1-\eta} - \overline{p}^{1-\eta}\right]$

On the other hand, from equation (17) and the definition of Ω_i we can write:

$$\begin{aligned}
|\Omega_i(p_A, p_A, p_B, p_B, F_A, F_B, x_i, \alpha_i)| &= |(1-\alpha_i)w_i(p_A, p_A, F_A, x_i) + \alpha_i w_i(p_B, p_B, F_B, 1-x_i)| \\
&< |w_i(p_A, p_A, F_A, x_i)| + |w_i(p_B, p_B, F_B, 1-x_i)| \\
&\leq W_i(p_A, p_A) + F_A + \tau x_i \sum_{\substack{j\in I \\ j\neq i}} \delta^{t_{ij}} + W_i(p_B, p_B) + F_B + \tau(1-x_i) \sum_{\substack{j\in I \\ j\neq i}} \delta^{t_{ij}} \\
&\leq I \frac{\left[\underline{P}_A^{1-\eta} + \underline{P}_B^{1-\eta}\right]}{\eta - 1} + 2\left[\overline{F} + \tau I\right] \\
&\leq 2\left[I \frac{\underline{P}^{1-\eta}}{\eta - 1} + \overline{F} + tI\right] \equiv N
\end{aligned}$$

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[18] If individual i is disconnected from the social network, then, whithout loss of generality, he can be removed from the set of consumers.

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